Symmetry Methods for Differential Equations: Exercise 1.4

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2013. 11. 12

Exercise (1.4). *Determine the value of α for which*

$$(x', y') = (x + 2\varepsilon, ye^{\alpha\varepsilon})$$

is a symmetry of

$$\frac{dy}{dx} = y^2 e^{-x} + y + e^x$$

for all $\varepsilon \in \mathbf{R}$.

Proof. The symmetry condition for the differential equation is

$$\frac{\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} w(x,y)}{\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} w(x,y)} = w(f(x,y), g(x,y)).$$

Where $w(x,y) = y^2 e^{-x} + y + e^x$,$f(x,y) = x + 2\varepsilon, g(x,y) = ye^{\alpha\varepsilon}$.So the symmetry condition can be written as

$$y^2 e^{-x+\alpha\varepsilon} + e^{x+\alpha\varepsilon} = y^2 e^{2\alpha\varepsilon - x - 2\varepsilon} + e^{x+2\varepsilon}.$$

So $\alpha = 2$. □